The mixing efficiency of interfacial waves breaking at a ridge: 
1. Overall mixing efficiency
E. L. Hult, C. D. Troy, and J. R. Koseff

1. Introduction

[1] The overall mixing efficiency of periodic, interfacial waves breaking at a Gaussian ridge is investigated through laboratory experiments. Cumulative measurements are used to investigate the fraction of the wave energy lost in the breaking event that contributes to irreversible mixing of the background density gradient. Using the tank as a control volume, the distribution of energy into reflected waves, transmitted waves, and dissipation and irreversible mixing from the breaking event is determined. The overall fraction of wave energy lost in the breaking event that is converted irreversibly to mixing is found to be 3–8%, which is low compared with typical values of around 20% for steady, parallel, stratified shear instabilities. Spatial variability in the mixing event may contribute to the relatively low overall efficiency of the event.


1. Introduction

[2] Internal wave breaking appears to contribute significantly to the vertical mixing of heat and mass in the ocean [e.g., Wunsch and Ferrari, 2004]. Thus, the efficiency with which these breaking events convert wave energy irreversibly to potential energy determines how much internal wave energy may be available for mixing the ocean. Understanding what governs this efficiency is critical to parameterizing smaller-scale internal wave events within larger-scale ocean models. It is very difficult to quantify the diapycnal diffusivity or the efficiency of mixing directly from ocean observations. Thus, in the interpretation of field data, it is typical to use a model that assumes a constant local mixing efficiency in order to infer the mixing from scalar spectra [Osborn, 1980; Oakey, 1982] despite a host of evidence from laboratory and numerical studies that the local mixing efficiency can vary significantly based on the applied forcing and stratification conditions [Ivey et al., 2008].

[3] The efficiency of internal wave breaking events is very difficult to measure directly in the ocean, and thus laboratory experiments and numerical simulations have been critical to studying the mixing efficiency of stratified flows. For first mode internal waves breaking on a slope in a linearly stratified fluid, the overall mixing efficiency reaches a maximum of 20% when the slope of the wave characteristic is equal to the beach slope [Ivey and Nokes, 1989]. For interfacial solitary waves breaking on a slope, the overall efficiency was found to have a maximum of 25%, varying with the length scale of the incident wave [Michallet and Ivey, 1999]. In the numerical simulations of Slinn and Riley [1996], critical rays focusing on a slope have a mixing efficiency of 35%. Interfacial wave breaking in deep water was considered by Fringer and Street [2003] who reported a peak overall efficiency of 36% ± 2% for highly nonlinear, progressive interfacial waves in numerical simulations and suggest the instability is convective, based on the high mixing efficiency. Although the mixing efficiency might be quite high during the laminar roll-up of a Kelvin-Helmholtz instability in a stratified shear flow, most of the mixing occurs after the transition to turbulence at an efficiency near 20%, based on the results of numerical simulations [Smyth et al., 2001; Peltier and Caulfield, 2003].

[4] The mixing efficiency in a stratified flow can vary with the instability mechanism. Purely convective instabilities may have a mixing efficiency of up to 50% [Linden and Redondo, 1991], whereas the mixing efficiency for parallel, steady shear flows is lower, typically about 20% [e.g., Peltier and Caulfield, 2003]. In part because of this significant difference in efficiency, there has been much interest in whether internal wave breaking is driven primarily by shear or convection. Hult et al. [2009] examine the qualitative nature of instability for periodic, interfacial waves breaking at a ridge and found that until the scaled amplitude over the ridge, a/h_r, is increased above a critical value, waves are initially stable. At a higher a/h_r, the waves exhibit backward breaking, and at yet higher a/h_r, waves steepen, plunge forward and then become gravitationally unstable. For sufficiently long, high-amplitude waves shear instabilities can contribute to wave breaking at the ridge, as well. Because the qualitative mechanism of the wave instability appears to depend on the scaled amplitude, a/h_r, it seems feasible that the overall efficiency with which wave energy is converted irreversibly to potential energy may depend on the scaled amplitude as well. In this study, the overall mixing efficiency

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is investigated for interfacial waves breaking at a submerged ridge for a range of incident wave amplitudes.

While the overall efficiency of interfacial waves breaking at a submerged ridge has not been measured previously in the lab, a number of studies have considered the reflection, transmission and dissipation of wave energy in such interactions. One parameter that has been used to distinguish the strength of wave obstacle interactions in two-layer flows is the degree of blocking

\[ B = \frac{h_0}{h_{1,\infty}}; \]

where \( h_{1,\infty} \) and \( h_0 \) are defined in Figure 1b. Internal solitary waves passing over a triangular obstacle are essentially not affected by the obstacle for \( B < 0.6 \), whereas waves split into a dispersive train when \( B < 0.8 \), and when \( B > 1.2 \) the obstacle blocks any transmission passed the obstacle [Wessels and Hutter, 1996]. Sween et al. [2002] introduce a blocking parameter, \( \zeta = (a + h_z)/(h_2 + h_r) \), to characterize transmitted internal solitary wave energy over a triangular obstacle. Chen [2009] reports that internal solitary wave reflection and energy loss scale monotonically with \( \zeta \). Wessels and Hutter [1996] considered the energy budget for internal solitary waves interacting with a ridge and found that up to 55% of the wave energy was dissipated in the wave-ridge interaction, and the dissipated fraction is maximum when \( 0.8 < B < 1.1 \). Chen et al. [2008] investigated the breaking of internal solitary waves at two sequential triangular obstacles in the laboratory and report that the fraction of incident wave energy lost over the obstacles is between 12% and 48%, increasing with the energy of the incident wave. The current study incorporates measurements of the change in background potential energy in addition to the wave energy loss, so that the overall event efficiency can be estimated for the wave-ridge interaction.

In this study, the change in potential energy is measured in addition to wave reflection, transmission and energy loss associated with the breaking event, so that the overall efficiency of the breaking event can be estimated. In this paper, the experimental setup is presented in section 2, followed by results on reflection and transmission of wave energy, potential energy and wave work measurements, and the overall efficiency in section 3. Section 4 is a discussion of the results and finally conclusions appear in section 5.

2. Experimental Setup

Experiments were performed in a 480 cm long by 30 cm wide by 60 cm tall tank with a vertically oscillating semicylindrical wavemaker at one end and a horsehair beach at the other end to prevent reflection from the back wall (see Figure 1a). The stratification consisted of two homogenous...
energy associated with the periodic waves propagating along the density interface. As the waves break at the ridge, some of the wave energy is converted to turbulent kinetic and potential energy, while some of the wave energy continues to propagate as waves, either reflected back from or transmitted past the ridge. Of the energy in the turbulence, some will be converted irreversibly to potential energy via molecular mixing hastened by turbulent straining. The rest of the turbulent energy eventually will be dissipated as heat. Energy propagating away from the ridge as waves will eventually be primarily dissipated as heat, as the waves are damped by the horsehair beach and the tank walls. Over time, some fluid will also tend to diffuse across the density interface, thickening this interface and converting the heat (internal energy) of the fluid irreversibly to potential energy. Once the fluid in the control volume of the tank has come to rest, the final potential energy can be compared with the initial potential energy to indicate how much mixing occurred during the experiment. Viscous effects and background diffusion are accounted for in the calculations of efficiency.

The procedure for the overall event efficiency experiments, adapted from Troy [2003], is as follows. First, an initial density profile was established at \( t = 0 \) s, using conductivity-temperature probe plunging vertically at 10 cm s\(^{-1}\). Next, starting at \( t = 30 \) s a train of waves was generated by the wavemaker. Sinusoidal oscillations of the wavemaker were increased from zero amplitude to the maximum amplitude over 1 wave period, then 10 waves were generated at the maximum amplitude, and finally the amplitude was reduced to zero over 1 wave period. During this time, images were taken at either upstream of the ridge at \( x_d = -65 \) cm or downstream at \( x_d = 85 \) cm. Although ideally measurements would have been taken upstream and downstream simultaneously for each experimental run, constraints on the imaging equipment allowed interfacial displacement measurements at only one station at a time. The variation between experiments was quite small (see Figures 3 and 6), so the error associated with comparing upstream and downstream interfacial displacement measurements from different experiments is thought to be minimal. Finally, after motions in the tank subside, a final density profile is obtained at \( t = 630 \) s. The mixing induced by the motion of the probe was negligible relative to background diffusion in the tank. This procedure was repeated for \( a_o \), between 0 and 4.4 cm, with 2 to 6 (typically 4) repetitions for each wave amplitude, providing 2 to 6 pairs of initial and final density profiles and one to three interfacial displacement records at the upstream and downstream locations. The wave amplitudes are shown in Table 1.

Table 1. Experimental Wave Conditions

<table>
<thead>
<tr>
<th>Stroke (cm)</th>
<th>( a_o ) (cm)</th>
<th>( a_o/h_2 )</th>
<th>( \zeta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0</td>
<td>0</td>
<td>0.84</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4</td>
<td>0.07</td>
<td>0.85</td>
</tr>
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<td>0.8</td>
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<tr>
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<td>1.2</td>
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<td>0.87</td>
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<tr>
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</tr>
<tr>
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<td>1.9</td>
<td>0.33</td>
<td>0.89</td>
</tr>
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<td>0.39</td>
<td>0.90</td>
</tr>
<tr>
<td>3.5</td>
<td>2.6</td>
<td>0.44</td>
<td>0.91</td>
</tr>
<tr>
<td>4.0</td>
<td>2.9</td>
<td>0.49</td>
<td>0.92</td>
</tr>
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<td>0.59</td>
<td>0.93</td>
</tr>
<tr>
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<td>0.65</td>
<td>0.94</td>
</tr>
<tr>
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<td>4.1</td>
<td>0.71</td>
<td>0.95</td>
</tr>
<tr>
<td>8.0</td>
<td>4.4</td>
<td>0.76</td>
<td>0.96</td>
</tr>
</tbody>
</table>

*Here, stroke is the wavemaker stroke and \( \zeta = (h_1 + a_o)/(h_2 + h_4) \). For all cases, \( h_{1,r} = 26 \) cm, \( h_2 = 30 \) cm and density \( \rho_1 = 1.0097 \) g cm\(^{-3}\), and the upper layer had depth \( h_2 = 30 \) cm and density \( \rho_2 = 0.9996 \) g cm\(^{-3}\). The two-dimensional, Gaussian ridge was 20.2 cm tall and extends 100 cm or 4 standard deviations (\( \sigma \)) in length. The forced wave frequency was \( \omega = 0.59 \) rad s\(^{-1}\) and the incident amplitude, \( a_o \), was varied between 0 and 4.4 cm, as shown in Table 1 with various nonlinearity parameters for comparison with other wave-ridge studies.

[s] Planar laser-induced fluorescence (PLIF) was used to image the flow during the experiments [e.g., Crimaldi, 2008]. Laser fluorescent dye (Rhodamine 6G) was added to the more dense, lower layer at a concentration of 50 ppb. An argon ion laser and a scanning mirror were used to generate a light sheet which was imaged by a 1024 × 1024 pixel CCD camera. The PLIF images were analyzed to track the movement of the interface, as this method was found to be more reliable than using ultrasonic interfacial wave gages, with the additional benefit of providing a record of the interfacial displacement across the field of view of the camera, in this case 19 cm wide. For additional details of the imaging setup and lab facility, see Troy and Koseff [2005].

[10] To assess the efficiency of wave breaking at converting wave energy irreversibly to potential energy through mixing, the change in wave energy and background potential energy due to the event must be calculated. In the laboratory, the entire wave tank can be used as a control volume for internal wave mixing experiments [Ivey and Nokes, 1989; Michallet and Ivey, 1999]. In the experiments in this study, energy enters the control volume via the oscillating wave maker which transfers energy to the fluid in the form of internal waves. There is both kinetic and potential layers separated by a thin interface. At the start of each experiment, the interfacial thickness was \( h = 1.5 \) cm ±\( 0.14 \) cm. The lower layer had depth \( h_{1,r} = 26 \) cm and density \( \rho_1 = 1.0097 \) g cm\(^{-3}\), and the upper layer had depth \( h_2 = 30 \) cm and density \( \rho_2 = 0.9996 \) g cm\(^{-3}\). The two-dimensional, Gaussian ridge was 20.2 cm tall and extends 100 cm or 4 standard deviations (\( \sigma \)) in length. The forced wave frequency was \( \omega = 0.59 \) rad s\(^{-1}\) and the incident amplitude, \( a_o \), was varied between 0 and 4.4 cm, as shown in Table 1 with various nonlinearity parameters for comparison with other wave-ridge studies.

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This estimate requires that local changes in depth be small relative to the wavelength, that energy is propagating in one direction, and that the Boussinesq approximation is valid. Because a small fraction of incident wave energy does reflect back from the ridge as discussed above, the upstream measurement location was chosen to coincide with a spatial node in the primary wave amplitude. At the node, the reflected component destructively interferes with the incident wave signal, so the amplitude measured at that location represents only the nonreflected portion of the wave energy, \( a_\infty = (a_{\text{incident}}^2 - a_{\text{reflected}}^2)^{1/2} \).

The total measured change in wave energy between the upstream and downstream measurement stations is shown in Figure 3 and can be expressed, \( \Delta W E_{\text{total}} = \Delta W E_{\text{break}} + \Delta W E_{\text{visc}} \). Here \( \Delta W E_{\text{total}} = W E_{\text{upstream}} - W E_{\text{downstream}} \) and \( \Delta W E_{\text{break}} \) refers to the wave energy change due to the breaking event, \( \Delta W E_{\text{visc}} \) is the wave energy lost to viscous decay not associated with the breaking event, i.e., through viscous damping at the tank boundaries as well as within the wave interface and interior, away from the breaking event [Troy and Koseff, 2006]. In order to calculate \( \Delta W E_{\text{break}} \), the \( \Delta W E_{\text{visc}} \) term must be estimated. Several methods are used here to estimate the nonbreaking, viscous losses in between the upstream and downstream stations. One method is to remove the ridge from the tank and repeat the experiments so that \( \Delta W E_{\text{visc}} \approx \Delta W E_{\text{total}} \) since there is no breaking. The change in wave energy for the ridge-less case is shown in Figure 3. It is important to note that the ridge-less case provides an underestimate of the nonbreaking losses, because the absence of the ridge increases the lower layer depth. Thus the wave velocities near the bottom are decreased, resulting in decreased viscous losses at the bottom and sidewalls. The ridge-less case results are shown for reference to illustrate the magnitude of losses away from the ridge.

A second method to estimate \( \Delta W E_{\text{visc}} \) is to use the theory developed by Troy and Koseff [2006] to estimate the viscous losses due boundary damping, internal dissipation, and interfacial dissipation. Figure 4 shows the expected variation in the wave amplitude over the ridge assuming linear wave shoaling and viscous decay [Troy and Koseff, 2006]. Away from the ridge, the dominant contribution to viscous wave decay is sidewall damping, and only over the very crest of the ridge does bottom boundary layer damping becomes dominant. The linear, viscous decay theory predicts the wave energy will decay by 18% without the ridge or 22% with the
ridge between the upstream and downstream measurement stations. The viscous damping estimates are also shown in Figure 3, where the viscous theory agrees reasonably well with $\Delta W_{E_{\text{visc}}}$ measured in the ridge-less case.

[15] A final method for estimating $\Delta W_{E_{\text{visc}}}$ was developed by Troy [2003]. In this method, $\Delta W_{E_{\text{visc}}}$ is extrapolated from the total change in wave energy for nonbreaking waves. In Figure 3, a quadratic curve is fit to the non-breaking wave points using least squares. Because viscous wave energy losses scale with the square of the wave velocity, $(a_w)^2$, a quadratic curve with an $a^2$ dependence is chosen. This quadratic curve is then extrapolated to give an estimate of viscous losses for the breaking waves. In Figure 3, $\Delta W_{E_{\text{visc}}}$ increases much more steeply for the breaking wave cases. The estimate of $\Delta W_{E_{\text{visc}}}$ from this extrapolation method is slightly higher than either the ridgeless case estimate or the linear viscous decay theory estimate.

[16] The three methods for estimating $\Delta W_{E_{\text{visc}}}$ give similar results, as shown in Figure 3. Because viscous losses increase where the ridge restricts the lower layer depth, it is expected that the no ridge case would lead to a lower $\Delta W_{E_{\text{visc}}}$. The viscous decay theory is linear, and thus it is not surprising that the associated estimate of $\Delta W_{E_{\text{visc}}}$ is slightly less than the extrapolation method estimate. For these reasons, the extrapolation method is used in final estimate of $Ri_{eq}$, and it is shown later that the viscous theory estimate yields very similar results. The reason for including all three methods is simply to support the extrapolation method and offer alternative methods for use when extrapolation is not feasible.

2.2. Change in Potential Energy Measurements

[17] If the tank is quiescent, the potential energy at an instant can be calculated from the density profile

$$\rho(z) = \rho_0 - \frac{\rho_1 - \rho_2}{2} \text{erf}(\beta z),$$

where $\rho_0 = (\rho_1 + \rho_2)/2$ and the length scale $\beta^{-1}$ can be related to the 99% interfacial thickness, $\delta = 3.64/\beta$. With the density profile in this form, equation (4) is evaluated analytically to give $\Delta PE$ in terms of initial and final interfacial length scales $\beta_{\text{initial}}$ and $\beta_{\text{final}}$

$$\Delta PE = gA \frac{\rho_1 - \rho_2}{2} \left( \frac{\beta_{\text{initial}}^2 - \beta_{\text{final}}^2}{2\beta_{\text{initial}}\beta_{\text{final}}} \right).$$

Using this method, the measured $\Delta PE$ is very consistent between repeated experiments. The change in potential energy between the initial and final density profiles is shown in Figure 6, where the maximum and minimum values of $\Delta PE$ are shown by the error bars.

[18] To estimate the changes in background potential energy that are not associated with the breaking event, the experimental procedure was repeated for each amplitude after removing the ridge from the tank, as discussed above.

**Figure 5.** Sample profiles of density, before (solid curve) and after (dashed curve) a train of breaking waves where $a_w/h_r = 0.59$.

**Figure 6.** Change in potential energy as a function of wave amplitude, $a_w$. $\Delta PE_{\text{total}}$ is shown for for breaking waves (open circles), nonbreaking waves (solid circles), and the no ridge case (crosses). $\Delta PE_{\text{nonbreak}}$ is extrapolated using a quadratic fit (solid curve).
for the estimation of $\Delta W_{E_{visc}}$. The change in potential energy for the ridge-less cases is shown in Figure 6. These results indicate that wave trains propagating in the absence of the ridge do not cause significant irreversible mixing above the level for the zero amplitude wave case. Therefore, mixing at the wavemaker and the horsehair beach must be minimal and the measured $D_{PE}$ in the no ridge case is assumed to be caused primarily by background diffusive processes over the time of the experiment. For unknown reasons, the variation between experiments appears to be greater in the ridge-less cases than when the ridge was in place, as indicated by both the $D_{PE}$ and $D_{WE}$ measurements.

In the cases where the ridge is in place, not all of the measured $D_{PE}$ is thought to be associated with the breaking event. A certain portion of the measured $D_{PE}$ is associated with background diffusive processes, as mentioned above. There is also expected to be a portion of $D_{PE}$ resulting from enhanced mixing due to the increased wave velocities over the ridge, as the wave amplitude is increased. For example, as the wave velocity increases, mixing is also expected to increase where the density interface intersects the boundary layer of the tank wall. This trend is shown in the slight increase of $D_{PE}$ for nonbreaking wave cases. As in the $D_{WE}$ measurements, the associated change in potential energy from boundary effects is expected to scale with $a^2$.

The total measured change in potential energy can be separated into breaking and nonbreaking components: $D_{PE_{total}} = D_{PE_{break}} + D_{PE_{nonbreak}}$. To estimate $D_{PE_{nonbreak}}$ a quadratic curve with an $a^2$ dependence is fit using least squares to the $D_{PE}$ measurements for the nonbreaking cases (e.g., $a_\infty < 2$ cm). $D_{PE_{break}}$ is estimated by extrapolating the quadratic curve to higher amplitudes, and then subtracting the nonbreaking portion of $D_{PE}$ from $D_{PE_{total}}$.

### 2.3. Overall Event Efficiency Calculation

From the change in wave energy associated with the breaking event, $\Delta W_{E_{break}} = (W_{E_b} - W_{E_d}) - \Delta W_{E_{visc}}$, and the change in potential energy associated with the breaking event, $\Delta P_{E_{break}} = \Delta P_{E_{total}} - \Delta P_{E_{nonbreak}}$, the overall event efficiency can be calculated as

$$R_f = \frac{\Delta P_{E_{break}}}{\Delta W_{E_{break}}}.$$  

The overall efficiency results are discussed in section 3.2.

### 3. Results

Figure 7 shows density fields calculated from PLIF images for breaking events of three incident wave amplitudes. As the incident amplitude is increased, breaking becomes more vigorous and the resulting turbulent patch becomes larger in size. After the initial wave instability, the flow transitions to turbulence, and then eventually decays. The mixed fluid resulting from the turbulent event eventually spreads away from the point of breaking along the interface. In Figure 7a, the wave is just past the breaking threshold and breaks mildly backward, whereas Figure 7b shows a more vigorous forward breaking wave, and Figure 7c shows an even more vigorous breaking event. From the density fields in Figure 7, it is clear that the higher the amplitude, the more mixing occurs during the breaking event, as the local
interfacial thickness increases substantially in Figure 7c compared with sequence (Figure 7a). Quantitative evidence of this trend can be seen in section 2.2.

3.1. Wave Reflection and Transmission

[22] For each incident wave, some energy is reflected back from the ridge. When a portion of the incident wave energy reflects back from the obstacle, the result is a spatial modulation of the wave amplitude upstream of the obstacle, as the reflected wave constructively and destructively interferes with the incident wave [e.g., Dean and Dalrymple, 1984]. The coefficient of reflection is typically defined as the ratio of the reflected wave amplitude and the incident wave amplitude, $C_R \equiv a_{\text{reflected}}/a_{\text{incident}}$. Here, $C_R$ can be estimated from the variation of the measured wave amplitude in space

$$C_R = \left(1 - \frac{a_{\text{mean}}^2}{a_{\text{mean}}^2}\right)^{1/2},$$

where $a_{\text{min}}$ and $a_{\text{mean}}$ are the local minimum and mean measured amplitudes in $a(x)$. Spectral analysis of the interfacial height time series from PLIF images gives the amplitude $a(x)$ across 19 cm image windows. A sine curve is fit to $a(x)$ using least squares, and the mean amplitude is then $a_{\text{incident}}$. Note, the relevant amplitude to the breaking event, $a_\infty$, is considered to be the transmitted wave energy. The transmitted energy is equal to the incident energy minus the reflected portion and the wave energy is proportional to the square of the amplitude, thus: $a_\infty = \left(a_{\text{incident}}^2 - a_{\text{reflected}}^2\right)^{1/2}$. In Figure 8a, the reflection coefficient, $C_R$, decreases as the wave number, $kh_1$, is increased for waves of constant incident amplitude. The error bars reflect variation in the fitted parameters between two repetitions of the experiment. The oscillation of $C_R$ with $kh_1$, seen in Figure 8a may be consistent with surface wave reflection from a submerged obstacle [e.g., Mei and Black, 1969]. Previous work indicates that $C_R$ is also sensitive to the degree of blocking, $B$ [Wessels and Hutter, 1996], but $B$ is not varied in this study.

[23] Somewhat surprisingly, the reflection coefficient appears to be insensitive to the incident wave amplitude, as shown in Figure 8b, where $C_R = 0.30 \pm 0.05$. A reflection coefficient of $C_R = 0.30$ corresponds to approximately 9% of the incident wave energy being reflected back from the ridge for both breaking and nonbreaking waves. The fact that $C_R$ is indifferent to whether or not the wave is breaking suggests that wave reflection from the ridge slope may take place upstream of where the breaking event occurs at the ridge crest. It is possible that $C_R$ may be affected by breaking when waves are shorter relative to the depth (higher $kh_1$), or when the wave slope ($h_0/2\sigma$) is steeper, either of which would compress the horizontal region over which breaking and reflection occur. The range of $C_R$ for the periodic waves in this study corresponds reasonably well with the results for internal solitary waves of depression reported by Chen [2009], where $0.3 < C_R < 0.6$ for $0.8 < \zeta < 1.0$ (see Table 1 to relate $\zeta$ and $a_\infty/h_f$ in this study). It appears that for periodic, progressive interfacial waves, however, $C_R$ varies with the length scale of the wave, and the parameter $\zeta$ does not capture this dependence explicitly as Chen studied solitary waves where $a$ and $k$ are not independent.

[24] The incident wave energy can be partitioned into reflected, transmitted and dissipated components based on information from the upstream and downstream interfacial displacement signals, such as those shown in Figure 2. The amplitude of the downstream interface displacement in Figure 2b is noticeably smaller than the upstream displacements in Figure 2a. Higher-frequency oscillations are also visible in the downstream signal in Figure 2b. Section 3.2 investigates how much incident energy is dissipated in the breaking event and how much energy goes to irreversible mixing.

3.2. Overall Event Efficiency

[25] The overall event efficiency in Figure 9 ranges from 3% to 8% ± 1%. This assumes that the nonbreaking wave energy loss is estimated from the quadratic fit to the nonbreaking points, ‘\(\Delta W_{\text{visc}}\) quad fit’ in Figure 3. If the
estimate of $\Delta W_{E_{\text{visc}}}$ from the viscous decay theory of Troy and Koseff [2006] is used instead, the resulting values of $R_{f0}$ are typically lower by about 0.7% (Figure 9). The uncertainty is quite large for wave amplitudes just large enough to break ($a_\infty/h_r \approx 0.45$ in Figure 9), because the uncertainty in the measured quantities is on the order of the difference, $\Delta P_{E_{\text{break}}}$ or $\Delta W_{E_{\text{break}}}$. It is notable, however, that for $a_\infty/h_r > 0.55$, at least half of $\Delta W_{E_{\text{total}}}$ and $\Delta P_{E_{\text{total}}}$ results from the breaking event. Without such a strong signal from the breaking event, this method is less effective at determining the overall event efficiency [Troy, 2003].

3.3. Wave Energy Partitioning

With the results discussed in this study thus far, the end fate of all of the incident wave energy is determined. Figure 10 illustrates how much of the incident wave energy was reflected from the ridge, dissipated through laminar viscous decay, transferred to higher harmonics, transmitted past the ridge, converted irreversibly to potential energy and dissipated in the breaking event ($\epsilon_{\text{break}}$). The total transmitted energy begins to decrease from about 60% when waves begin to break at $a_\infty/h_r \approx 0.4$. Once waves begin to break, the total incident wave energy transmitted past the ridge decreases from about 60 to 30% and an increasing portion is dissipated in the breaking event. As discussed by Hult et al. [2010], waves of higher harmonic frequencies can be excited when a train of periodic waves passes over the ridge and thus the transmitted wave energy at twice the forcing frequency ($2\omega_1$) is shown separately from the rest of the transmitted wave energy. The energy transmitted at twice the forcing frequency ($2\omega_1$) peaks when $a_\infty/h_r \approx 0.4$. While a substantial fraction of the incident wave energy is dissipated in the breaking event, only a small percentage of the wave energy goes to irreversible mixing ($\Delta P_{E_{\text{break}}}$). The portion of energy dissipated during breaking appears to level off as $a_\infty/h_r$ approaches unity.

4. Discussion

The overall event efficiency of 3–8% is much lower than the 36% efficiency reported for deep water interfacial wave breaking [Fringer and Street, 2003; Troy, 2003]. It is reasonable that the overall efficiency of an event would be lower for a topography induced breaking event than for deep water wave breaking. In a deep water breaking event, both density overturning and turbulent kinetic energy dissipation are focused along the interface. When breaking occurs at a ridge or slope, on the other hand, the wave flow tends to be focused along the topographic boundary. A two-layer wave interaction with a topographic feature may, however, lead to separation vortices within the well-mixed lower layer that do little to alter the background density gradient. As seen in Figure 12 of Hult et al. [2009], strong flow over the ridge in the vertically constricted lower layer separates and the development of a large vortex is observed on the upstream side of the ridge as the wave crest approaches. The generation of this separation vortex could influence the overall efficiency of the wave-ridge interaction, as energy is transferred from the propagating wave to the vortex in a region where the background density gradient is extremely weak. This process may help to explain why the overall event efficiency is much lower than the deep water case. Hult et al. [2011, hereafter Part 2] use high-resolution

Figure 9. Overall event efficiency $R_{f0}$ as a function of scaled wave amplitude $a_\infty/h_r$, where $\Delta W_{E_{\text{visc}}}$ is estimated from the quadratic fit to the nonbreaking points (open circles) and from the viscous decay theory of Troy and Koseff [2006] (solid circles).

Figure 10. Partitioning of incident wave energy, where $\Delta W_{E_{\text{visc}}}$ is estimated from the quadratic fit method.
measurements over the spatial domain to explore this hypothesis of variation in local mixing processes.

[26] The range reported here is at the low end, compared with the 15 ± 5% efficiency reported by Helfrich [1992], or the 7–25% efficiency reported by Michallet and Ivey [1999], both for breaking solitary waves at a slope. Note that Michallet and Ivey did not account for background diffusive effects in their analysis, which would conservatively lower their efficiency range of 3–22%. If it is the interaction with topography causing spatial variability in the mixing process, why would the overall event efficiency in this study be so low compared with previous results for internal solitary waves breaking at a slope? While there is much in common with the slope case, there are some key differences in the flow for waves over a ridge. In the ridge case, there is strong, periodic flow over the ridge crest, including acceleration of the flow around the obstacle and separation from the boundary. This may lead to more energy dissipation within the lower layer than in the slope case where motion is concentrated where the density interface intersects the slope. The energy dissipation and mixing within the lower layer is discussed in Part 2.

[29] Although the range of overall mixing efficiency is quite low, the most nonlinear breaking events were 2.5 times more efficient than the least nonlinear cases. This difference may be due to the range of breaking mechanisms that occurred in this study. As reported by Hult et al. [2009], when 0.45 ≤ a∞/h ≲ 0.55, breaking events appear to be primarily driven by shear, whereas convective instability is observed when 0.55 ≤ a∞/h. The transition values depend on the scaled wavelength, and those listed here correspond with kδ = 0.11 as in this study. In Figure 9, when a∞/h, exceeds about 0.55, the overall event efficiency increases from 3% to 8%. The dependence of the overall efficiency on the topographic slope, wave frequency or interface height was not explored here.

[30] When the wave amplitude is small relative to the lower layer depth over the obstacle (a∞/h < 0.2), the dependence of energy transmission on the degree of blocking, B, developed by Wessels and Hutter [1996] appears to break down. Wessels and Hutter [1996] stated that waves were unaffected by the presence of a ridge when B < 0.6. However in this study, B is held constant at 0.78, and very low amplitude waves were hardly affected by the ridge whereas in high-amplitude cases significant energy was dissipated at the ridge (see Figure 10). This suggests that the amplitude of the incident wave affects the intensity of the wave-ridge interaction. While the parameter ζ indicates a dependence on amplitude, there is a discrepancy in the energy dissipated at the ridge between the results in Figure 10 and of Chen et al. [2008], likely due to the insensitivity of the wave-ridge interaction to the upper layer depth in the periodic wave case.

[31] While the distribution in Figure 10 provides an overview of how energy is transferred as a function of a∞/h, the percentages in the distribution may vary to some extent with additional wave and topographic parameters. The overall mixing efficiency may vary to some extent based on breaking mechanism, as discussed above. Although this limit was not explored in this study, it seems probably that the mixing efficiency would increase as h0 is reduced dramatically, and the geometry approaches the case of interfacial wave breaking on a slope case studied by Michallet and Ivey [1999]. Also, the fraction of the incident wave energy that goes to reflection, transmission, higher harmonics, viscous dissipation and the breaking event (ε + ΔPE) will vary to some extent with the wave and topographic parameters. The percent of incident energy transmitted for internal solitary waves passing over a ridge [Helfrich, 1992] has been shown to vary with the length scale of the wave. As seen in Figure 8a, the amount of energy reflected from the ridge tends to increase with the incident wavelength, whereas viscous decay tends to damp shorter waves more quickly. The wave nonlinearity, the strength of the stratification relative to the wave frequency, and the wavelength relative to the topographic length can affect the excitation of higher harmonics [Hult et al., 2010].

[32] It is possible that the sidewall or bottom boundary layers may become turbulent as the wave amplitude is increased. In this case, the choice of an a2 or the use of laminar viscous theory to estimate the nonbreaking wave energy loss may not be appropriate. Troy and Kosseff [2006] suggest the transition to turbulence occurs when the sidewall Reynolds number Rg = u∞/Prorbot/v is 200–800, where Lrobor is the orbital excursion distance. The conservative approximations for this case, u∞ ≈ aω, and Lrobor ≈ a, give Rg = 300–1200 for the breaking wave cases which suggests the sidewall boundary layers are probably turbulent for the highest-amplitude cases in this study. Over most of the tank length, the dominant contribution to viscous damping is from the sidewalls. Only as waves pass over the very crest of the ridge is damping at the bottom dominant. If a transition to turbulent sidewall boundary layers occurs, the viscous damping rate is expected to increase compared to the laminar rate. The overall event efficiency is not, however, terribly sensitive to the sidewall damping coefficient. If a transition to turbulence doubled the sidewall and bottom damping coefficients estimated by the laminar theory of Troy and Kosseff [2006], then Rg would only increase by about 2% to give Rg = 5–10%, assuming no change in ΔPE.

[33] The Reynolds number of these experiments may be modest relative to oceanic flows. The key message, however, is relevant to flows of a wide range of scales. If internal wave energy is dissipated in an interaction with topography, the overall event efficiency of the event may vary depending on the instability mechanism. The hypothesis to be explored in Part 2 that spatial variability in the stratification can reduce the overall event efficiency from a typical value of 20–25% is also expected to be independent of Re. The details of the separation flow may depend on the Reynolds number, however separation can occur over a wide range of Reynolds number [e.g., Kundu and Cohen, 2004].

5. Summary

[34] Using the interfacial displacement upstream and downstream of the wave-ridge interaction as well as the measured change in potential energy during the experiment, the partitioning of energy for periodic, progressive interfacial waves breaking at a ridge was investigated. Between 30% and 65% of the incident wave energy was transmitted over the ridge, where the fraction of energy transmitted
tends to decrease as the amplitude of the wave is increased. The reflection coefficient $C_R = a_{\text{reflected}}/a_{\text{incident}}$ was shown to decrease from 0.5 to 0.2 (i.e., 25% to 4% of incident wave energy) with scaled wave number, $kh_{1,0}$. When the amplitude is varied for a fixed wave number, $kh_{1,0} = 2.0$, the reflection coefficient remains constant at $C_R = 0.30 \pm 0.05$ for nonbreaking and breaking wave cases.

[35] The fraction of wave energy lost in the breaking event that is converted irreversibly to potential energy, referred to as the overall event efficiency $R_{fo}$, is estimated by using the wave tank as a control volume. The wave energy is measured upstream and downstream of the breaking event, and viscous losses not associated with the breaking event are estimated with several methods including extrapolating from nonbreaking wave cases and viscous decay theory [Troy and Koseff, 2006]. The overall event efficiency is between 3% and $8\% \pm 1\%$ for waves of varied incident amplitude. There is not a clear demarkation in overall efficiency between shear and convective breaking events, however, the efficiency tends to increase slightly with the nonlinearity $(a_{s}/h_{o})$. While this efficiency is consistent with results for breaking internal waves at topography [Ivey and Nokes, 1989; Helfrich, 1992; Michallet and Ivey, 1999], the measured efficiency of interfacial waves in deep water is as high as 36% [Fringer and Street, 2003; Troy, 2003]. The relatively low overall efficiency appears to be related to spatial variability in the mixing processes. The wave–ridge interaction can lead to separation of the lower layer flow over the ridge, which in turn can cause enhanced turbulent kinetic energy dissipation as shown in Part 2. In this region of enhanced dissipation within the essentially homogenous lower layer, the expected mixing efficiency would be quite low as there are no density gradients to mix, and this could reduce the overall mixing efficiency relative to the deep water case where turbulence is concentrated in the interfacial region. The impact of spatial variability on the overall efficiency is investigated in Part 2.

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References


